① Answer any Four questions : $5 \times 4 = 20$

② (i) what do you mean by a monotonic sequence ?

(ii) Prove that the sequence $\{\frac{1}{n}\}$ Converges to 0. $(2+3)$

⑥ Prove that the sequence $\{x_n\}$, where
$x_n = \left(1 + \frac{1}{n}\right)^n$ is Convergent.

© (i) State Cauchy's general principle of Convergence.

(ii) Use a Cauchy's general principle of Convergence to prove that the sequence $\{\frac{n}{n+1}\}$ is Convergent. $(2+3)$

④ (i) Examine the Convergence of $\sum\limits_{n=1}^{\infty} \frac{n^n}{n!}$

(ii) Show that the series $\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \cdots + \frac{n}{2^n}$ is Convergent. $(2+3)$

ⓔ Show that the maximum value of $x^x \log\left(\frac{1}{x}\right)$ is $\frac{1}{2e}$.

ⓕ (i) Evaluate $\lim\limits_{x \to 1} \left\{ \frac{x}{x-1} - \frac{1}{\log x} \right\}$

(ii) Evaluate $\lim\limits_{x \to 0} \frac{\sin x - x}{x^3}$ $(2+3)$

(2) Answer any three questions: $5 \times 3 = 15$

(a) Solve by the method of variation of parameters:

$$\frac{d^2y}{dx^2} + a^2 y = \sec ax.$$

(b) $(5+2x)^2 \frac{d^2y}{dx^2} - 6(5+2x)\frac{dy}{dx} + 8y = 8(5+2x)^2$

(c) Show that $[\alpha+\beta, \beta+\gamma, \gamma+\alpha] = 2[\alpha \beta \gamma]$

(d) Find the unit vector perpendicular to both
$\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$. Find also
the angle between them.

(3) Answer any two questions: $5 \times 2 = 10$

(a) Calculate $\gcd(567, 315)$ and
express $\gcd(567, 315)$ as $567 u + 315 v$.

(b) (i) Prove that $19^{20} \equiv 1 \pmod{181}$

    (ii) Find the least positive ~~red~~ residue
in $3^{36} \pmod{77}$ $(2+3)$

(c) (i) show that $a^{12} - b^{12}$ is divisible by 91 if
a and b are both prime to 91.

    (ii) If n be an odd positive integer prove
that $\phi(2n) = \phi(n)$ $(3+2)$